# Regime shifts and tipping points

# Some fundamentals.

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Outline:

- General ideas and Examples.
- Regime transitions as bifurcations.

# Terminology

Tipping points in climatology.

Regime shifts in ecology.

More in general (Condensed Matter Physics, Bifurcation theory Maths, …) Critical Transitions or Critical Points.

The name tipping point appeared first in the context of Sociology.

1) Dakos, Scheffer, van Nes, Brovkin, Petoukhov, Held,
Slowing down as an early warning singal for abrupt climate change,
PNAS 105, 14308 (2008)
2) Andersen, Carstensen, Hernández-García, Duarte, Ecological thresholds and regime shifts: Approaches to identification, Trends in Ecology and Evolution 24, 49 (2009).
3) Scheffer, Carpenter, Foley, Folke, Walker, Catastrophic shifts in Ecosystems, Nature 413 (2001).
4) Scheffer, Critical Transitions in nature and society, Princeton University Press, 2009.
5) Thompson and Sieber, Predicting Climate tipping as a noisy bifurcation: a review, International Journal of Bifurcation and Chaos 21, 399 (2011).
7) Thompson, Stewart, Ueda, Safe, explosive, and dangerous bifurcations in dissipative dynamical systems, PRE (1994)
8) Strogatz, Nonlinear Dynamics and Chaos.
9) Sole, Phase Transitions.

Tipping points:

- A small perturbation producing a large change/response in the system.

- Occur in many complex environmental systems.

- They produce abrupt and sometimes irreversible changes. They have high impact on the system.

- Inherently difficult to predict.

- Considerable challenges to occupants and managers of those systems.

**The phrase “tipping point” captures the notion that sometimes little things can make a big difference to the state and/or fate of a system.**

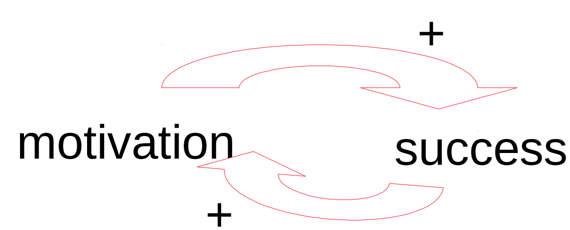
- Tipping points are often associated with systems exhibiting multiple stable states (alternative attractores) under the same conditions → transitions between them.

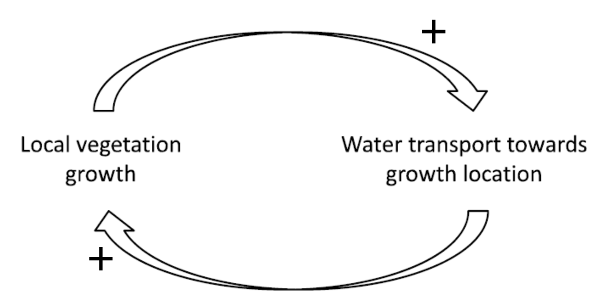
- For a system to exhibit a tipping point there must be **STRONG POSITIVE FEEDBACK** in its internal dynamics.

- Positive feedback means a closed loop of causal connections that are self-amplifying, tending to increase any perturbation.

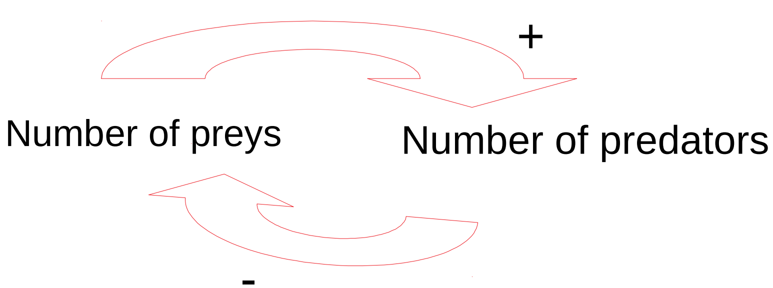
- If the feedback is strong enough (threshold behavior) for the perturbation to get larger after every loop. Such a mechanism can propel an abrupt transition between different attractors of the system.

**Positive feedback loop**



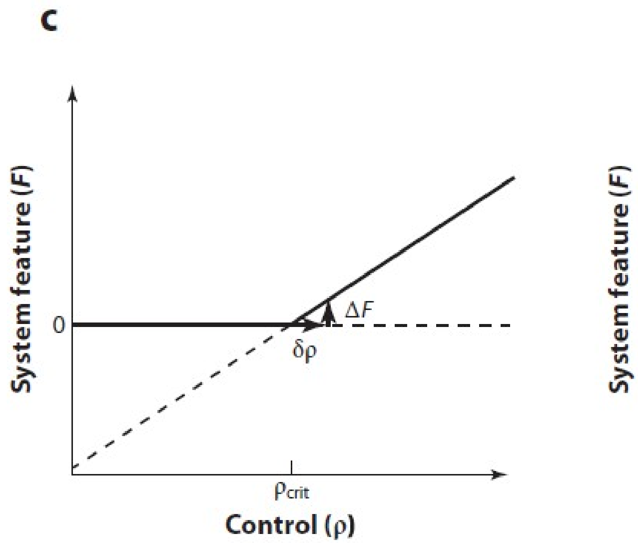

**Negative feedback loop**



- Of particular interest are the subset of catastrophic bifurcations.

- In fact, much of the current focus is on just one type of bifurcation: the "saddle-node" bifurcation (when there are two). Steady forcing past such a bifurcation point, where an attractor disappears, causes an abrupt and discontinuous transition to an alternative attractor.

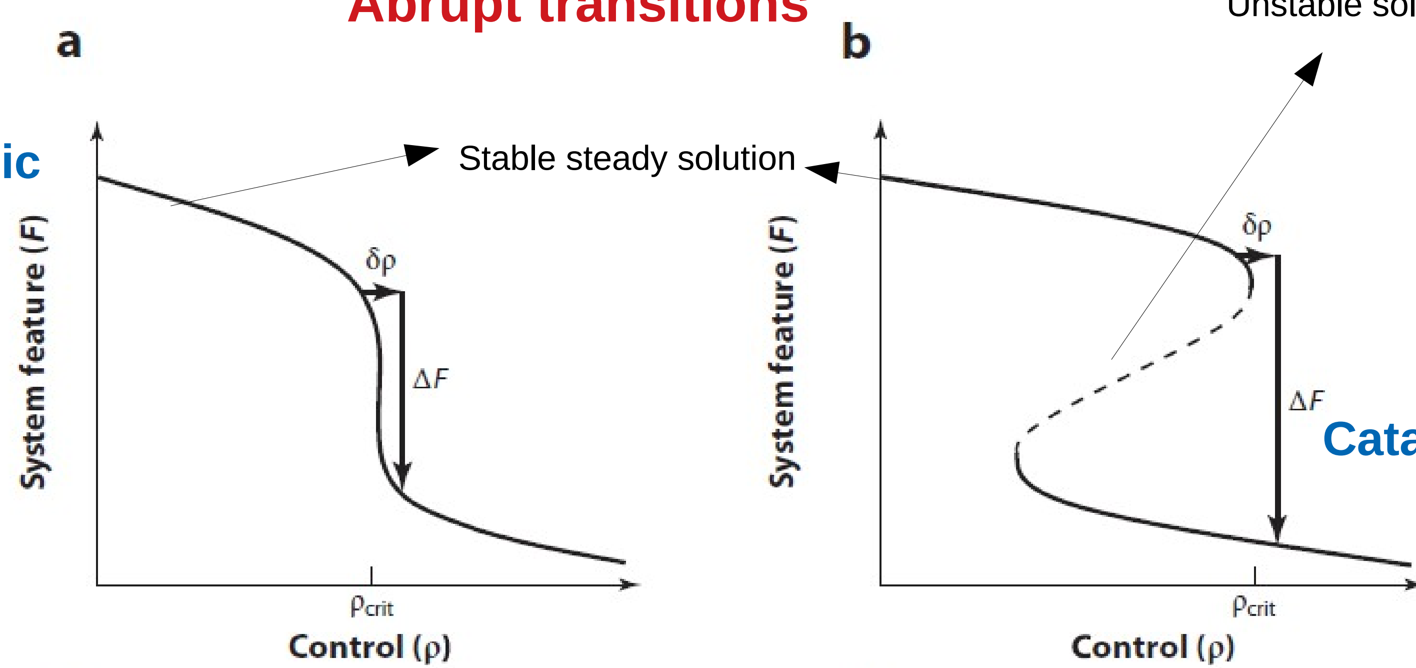
- Catastrophic involves irreversible changes which means that when the control parameter is returned to the threshold value ($\rho$ crit ) it does not revert to its original state. However, other non-catastrophic bifurcations and phase transitions are reversible at the same value of the control.




**Smooth transition**

**Abrupt transitions**

**Non Catastrophic**



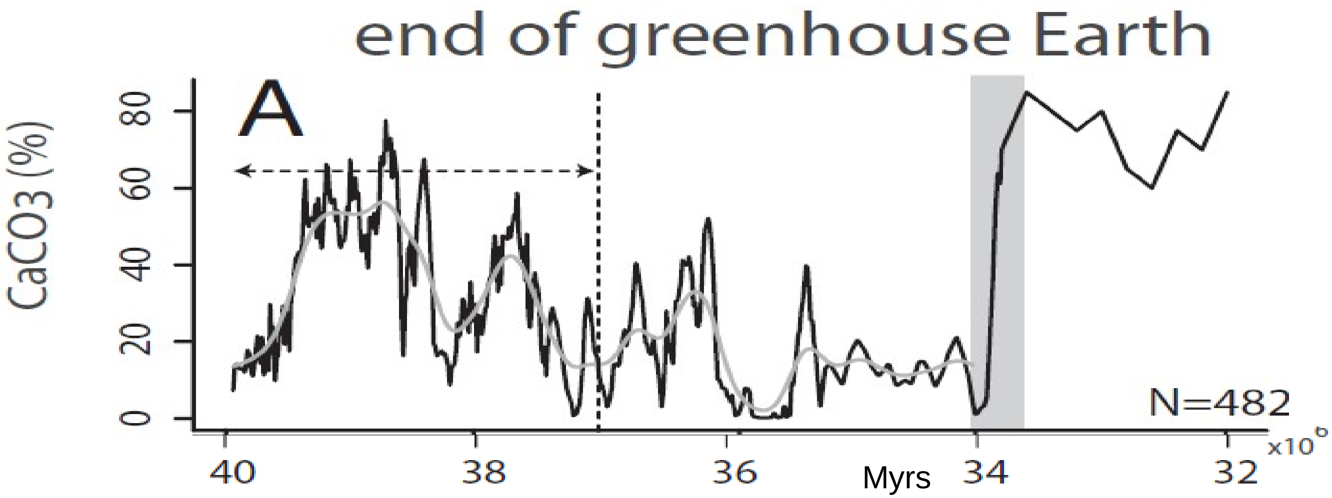

**Catastrophic**

**BISTABLE with saddle-node bifurcations**

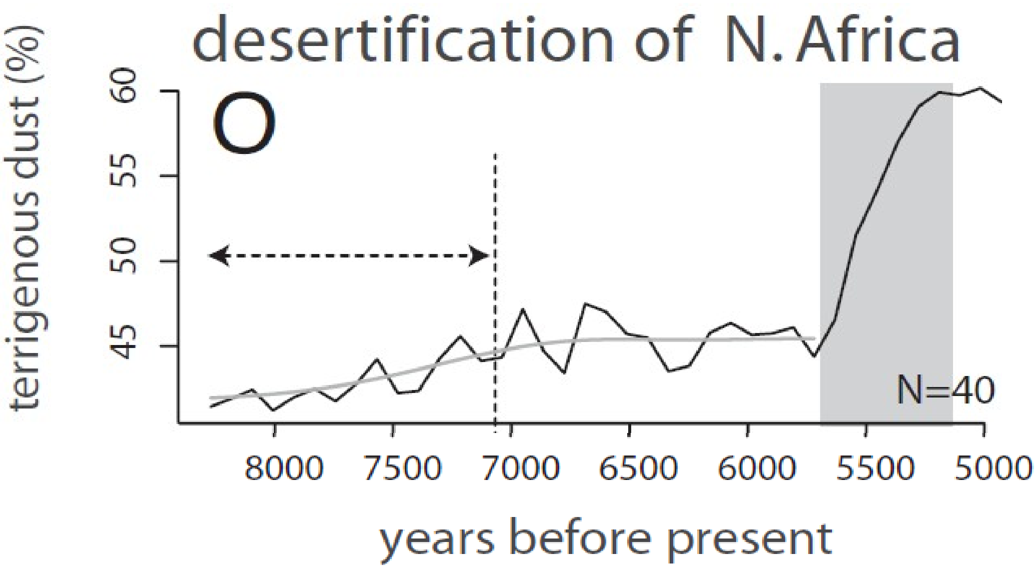
# Some climatic examples

In climate, despite its relative constancy over thousand years, one may find sharp transitions (at different time scales). Such studies have been made both on climatic computer models and on real paleoclimate data. The latter employ time-series
provided by ice cores, sediments, etc..

**From Dakos et al. PNAS 105, 14308 (2008)**



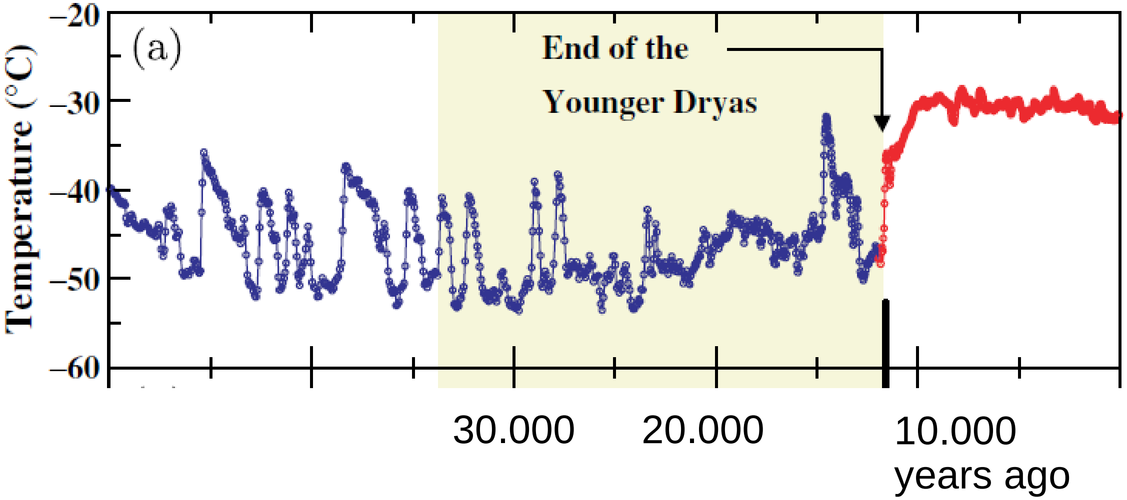

The greenhouse-icehouse transition: change from the tropic state to a state with ice caps (the two dominant climate states).




Desertification in North Africa: from a savanna like state to a desert state.

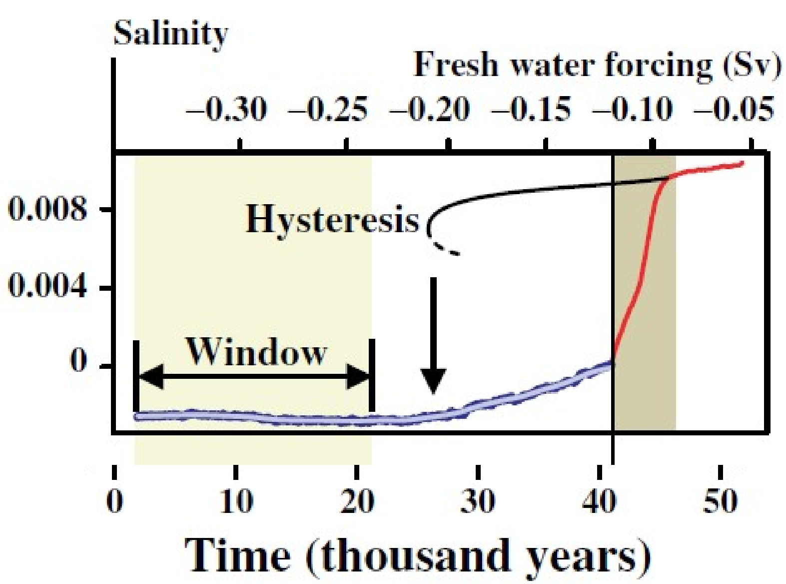
**From Dakos et al. PNAS 105, 14308 (2008)**

**End of last glaciation: using ice-core paleo-temperatures**




Younger Dryas period occurred just after the recovery from the last glacial maximum, the climate at Greenland relapsed to very cold conditions for many centuries and then suddenly jumped back to a 10° warmer state. The artic warmed ~ 7 degrees in 50 years.

**Collapse of Thermohaline Circulation**




Simulated abrupt climate transitions.

**From Dakos et al. PNAS 105, 14308 (2008)**

## Tipping elements in the Earth's climatic system:

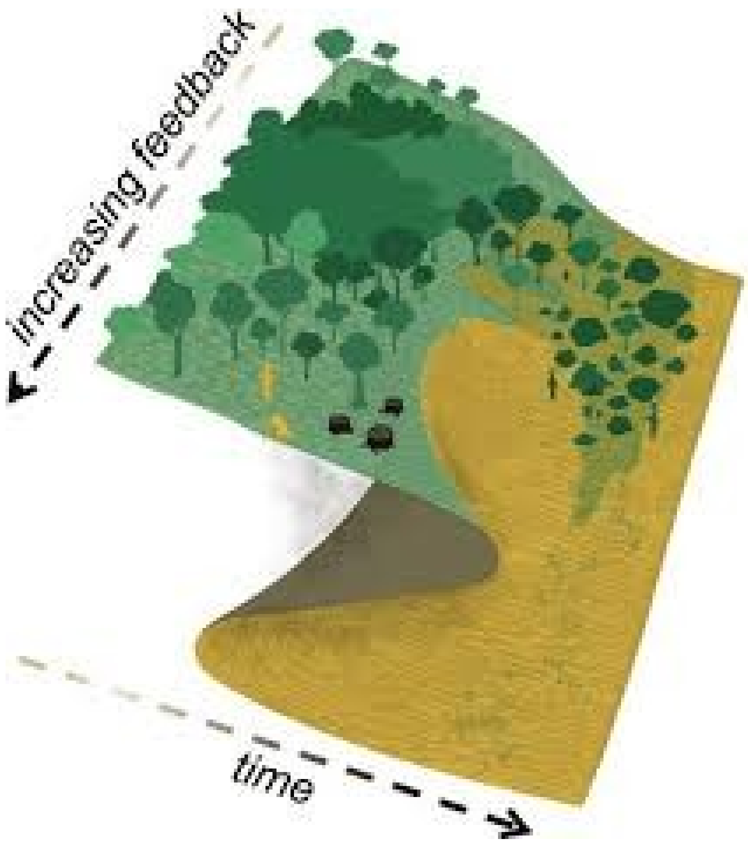
## large-scale components (subsystems) of the Earth system that may pass a tipping point.

**Summary of Lenton et al. (PNAS 105, 1786 (2008)) TIPPING ELEMENTS** (From Thompson and Sieber, IJBC 21, 399 (2011))

| Tipping Element | Feature, $F$ (Change) | Control Parameter, $\mu$ | $\mu_{crit}$ | Global Warming | Transition Time, $T$ | Key Impacts |
|---|---|---|---|---|---|---|
| Arctic summer sea-ice | Areal extent (−) | Local $\Delta T_{air}$, ocean heat transport | ?? | +0.5 to +2°C | ~10 yrs (rapid) | Amplified warming, ecosystem change |
| Greenland ice sheet (GIS) | Ice volume (−) | Local $\Delta T_{air}$ | ~ + 3°C | +1 to +2°C | >300 yrs (slow) | Sea level +2 to +7 m |
| West antarctic ice sheet (WAIS) | Ice volume (−) | Local $\Delta T_{air}$ or, less $\Delta T_{ocean}$ | +5 to +8°C | +3 to +5°C | >300 yrs (slow) | Sea level +5 m |
| Atlantic thermohaline circulation | Overturning (−) | Freshwater input to North Atlantic | +0.1 to +0.5 Sv | +3 to +5°C | ~100 yrs (gradual) | Regional cooling, sea level, ITCZ shift |
| El Niño Southern oscillation | Amplitude (+) | Thermocline depth, sharpness in EEP | ?? | +3 to +6°C | ~100 yrs (gradual) | Drought in SE Asia and elsewhere |
| Indian summer monsoon (ISM) | Rainfall (−) | Planetary albedo over India | 0.5 | N/A | ~1 yr (rapid) | Drought, decreased carrying capacity |
| Sahara/Sahel and W. African monsoon | Vegetation fraction (+) | Precipitation | 100 mm/yr | +3 to +5°C | ~10 yrs (rapid) | Increased carrying capacity |
| Amazon rain-forest | Tree fraction (−) | Precipitation, dry season length | 1100 mm/yr | +3 to +4°C | ~50 yrs (gradual) | Biodiversity loss, decreased rainfall |
| Boreal forest | Tree fraction (−) | Local $\Delta T_{air}$ | ~ + 7°C | +3 to +5°C | ~50 yrs (gradual) | Change in type of the ecosystem |

**These are all large scale phenomena. In CAFE we are interested in shorter scales ...**

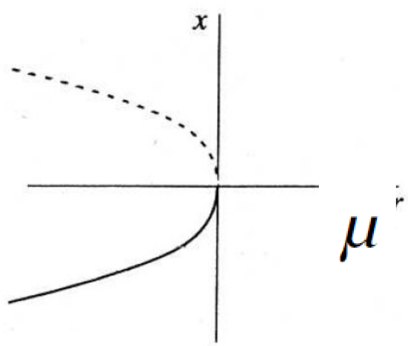
Transitions as bifurcations (=qualitative change on the number or type of the attractors as a parameter is changed)



# Some ideas of bifurcation theory for one-dimensional ODEs

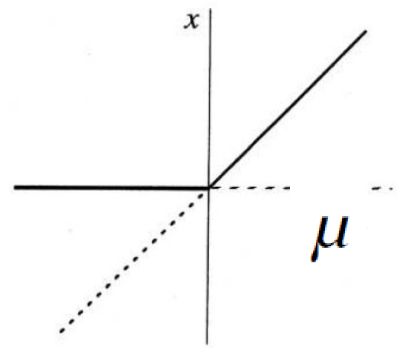
$$\dot{x} = f(x). \qquad\qquad f(x) = -\frac{dV}{dx}.$$

Fixed point *x* / f(x*)=0*
*Stable f'(x*) <0*
*Unstable f'(x*)>0*
*Neutral or bifurcation point if f'(x*)=0*



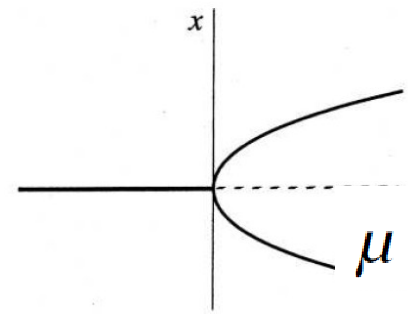

Saddle-node

$$\dot{x} = \mu - x^2$$



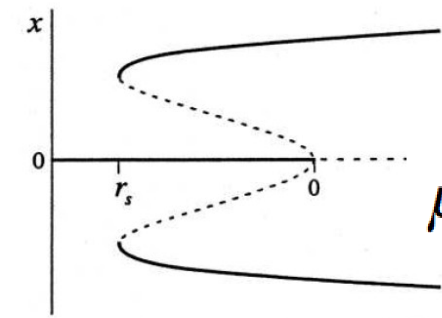

Transcritical

$$\dot{x} = \mu x - x^2$$




Supercritical Pitchfork

$$\dot{x} = \mu x - x^3$$



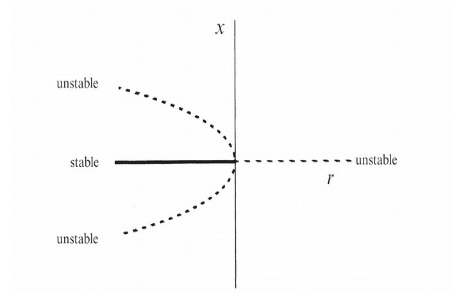

Subcritical Pitchfork

$$\dot{x} = \mu x + x^3 - x^5$$

A stable/unstable pair of fixed points is created or destroyed (also called fold bifurcation).

A fixed point interchanges its stability with another

A stable f.p. becomes stable, and new stable are created



# Some classifications

- **Supercritical vs subcritical:**

**Supe**r: a stable branch of solutions existing for $\mu < \mu_c$ becomes unstable leading to an unstable branch of solutions. **Sub**: a stable branch of solutions, for $\mu < \mu_c$ , becomes unstable past the bifurcation point after colliding with an unstable branch of solutions that exists for $\mu < \mu_c$ . So, in practical terms, in both cases a stable branch of solutions, for $\mu < \mu_c$ , leads to a unstable one, for $\mu > \mu_c$ , the difference being that in the supercritical case a stable branch of solutions appears for $\mu > \mu_c$, while in the subcritical case a unstable branch of solutions for $\mu < \mu_c$ disappears at the bifurcation point.

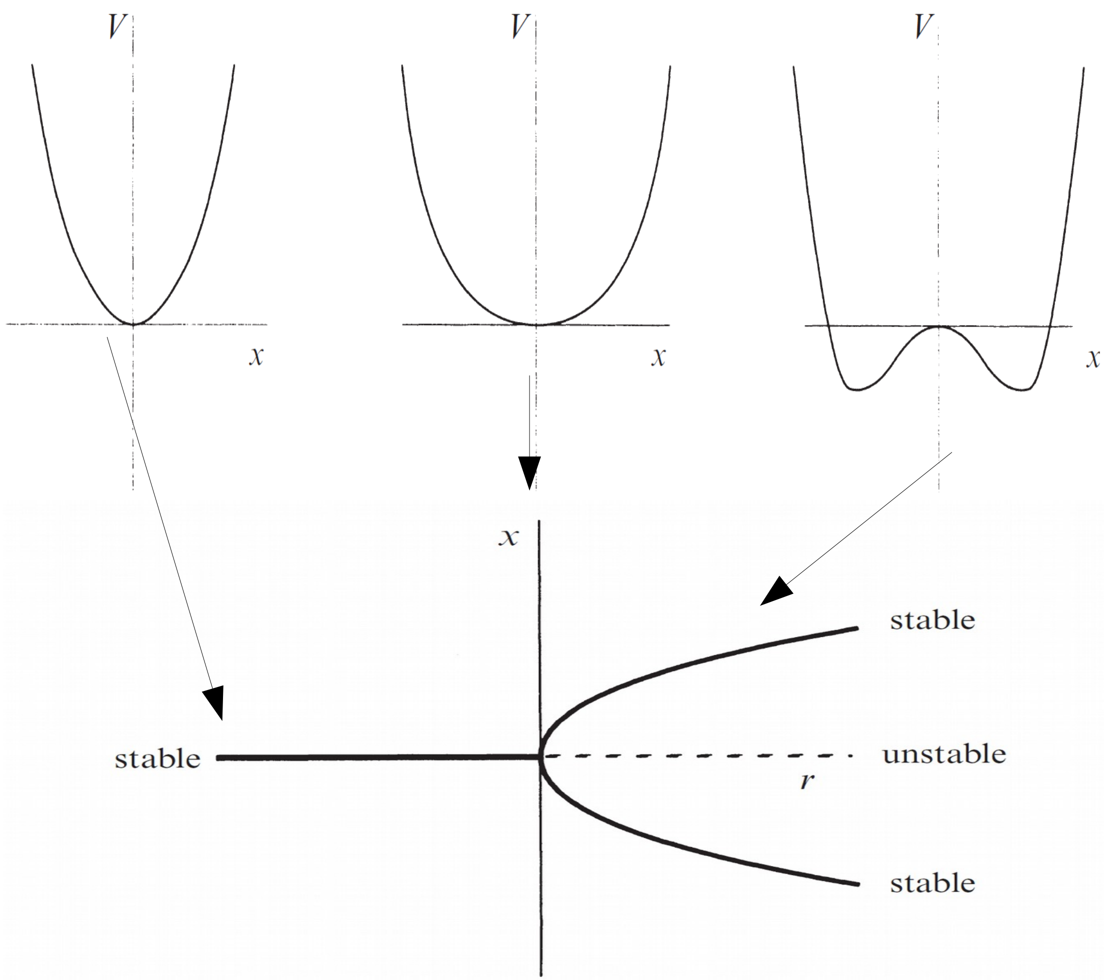
- **Local vs global**: local changes close to the fixed point vs global changes in the phase space.

- **Showing Intermittency or hysteresis**: Intermittency is the irregular alternation of phases of apparently periodic and chaotic dynamics, or different forms of chaotic dynamics.

# Supercritical pitchfork vs subcritical pitchfork: Safe vs Dangerous

## Supercritical

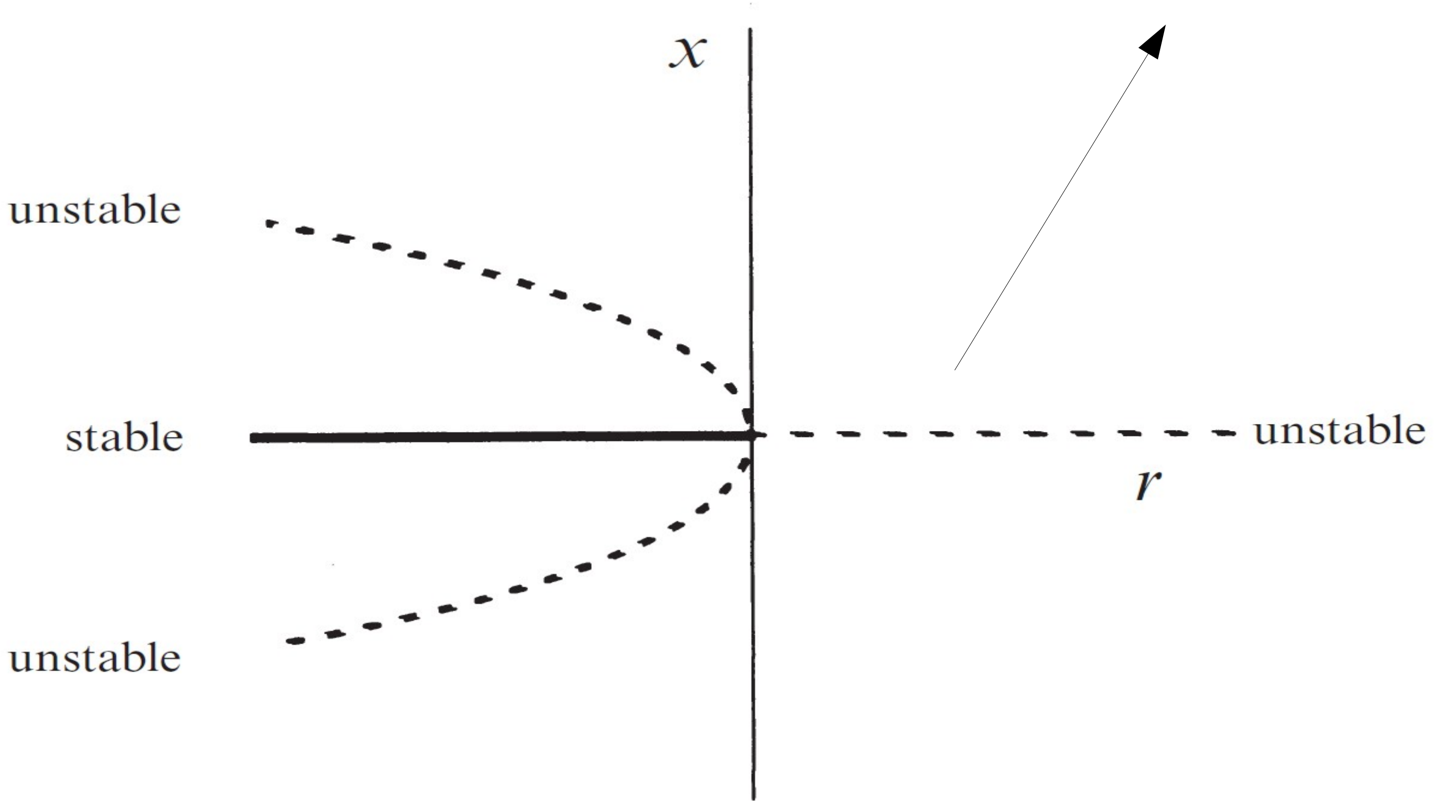
$$\dot{x} = rx - x^3. \qquad V(x) = -\tfrac{1}{2}rx^2 + \tfrac{1}{4}x^4 \qquad -dV/dx = rx - x^3$$

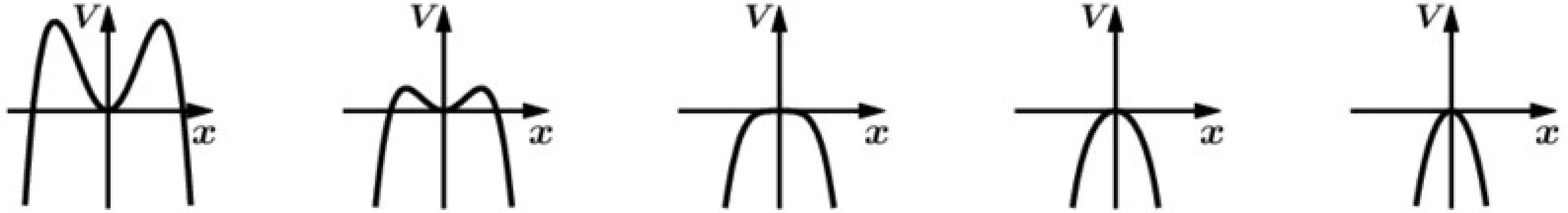


## Subcritical:

$$\dot{x} = rx + x^3$$

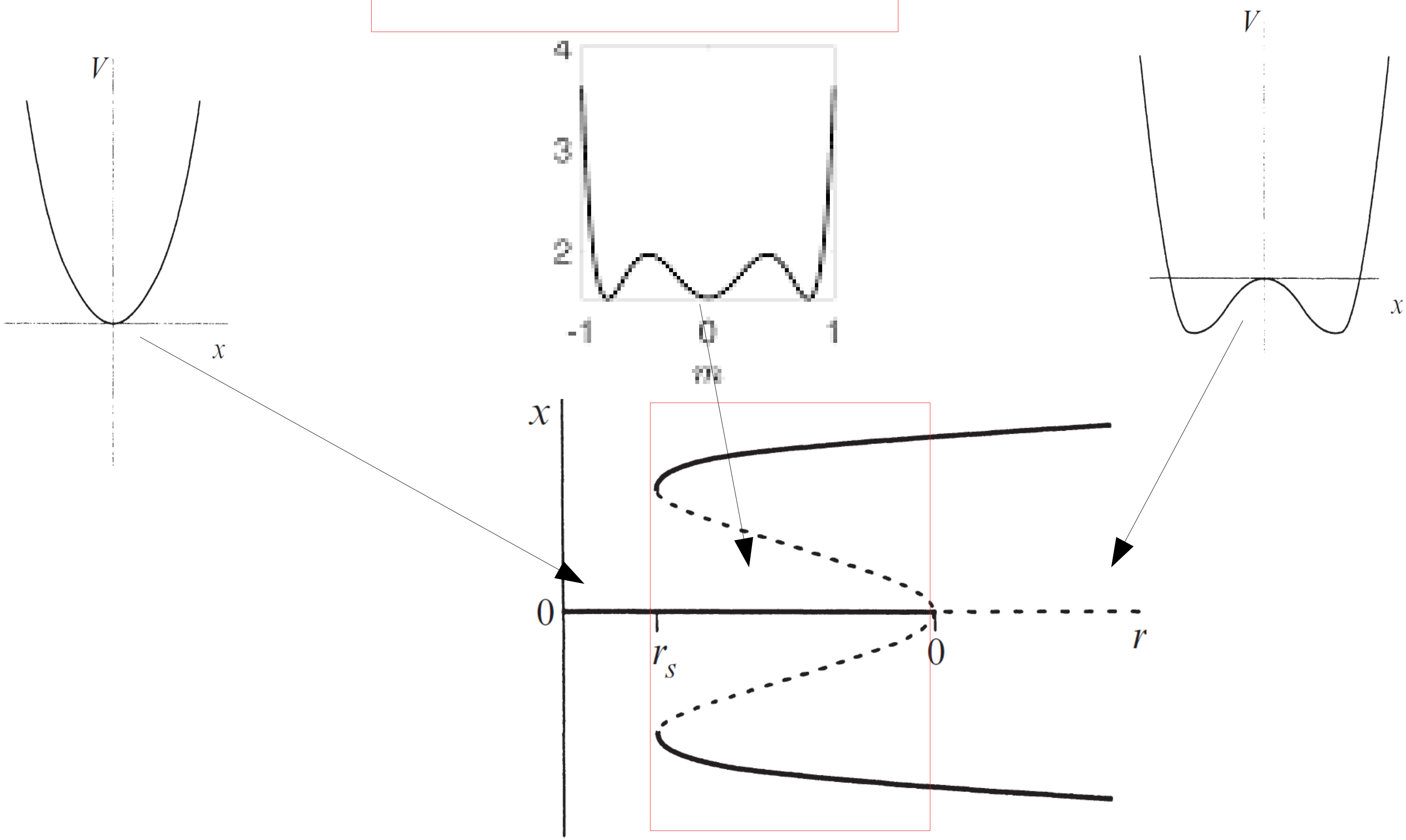
Dynamical system not well-defined for r>0



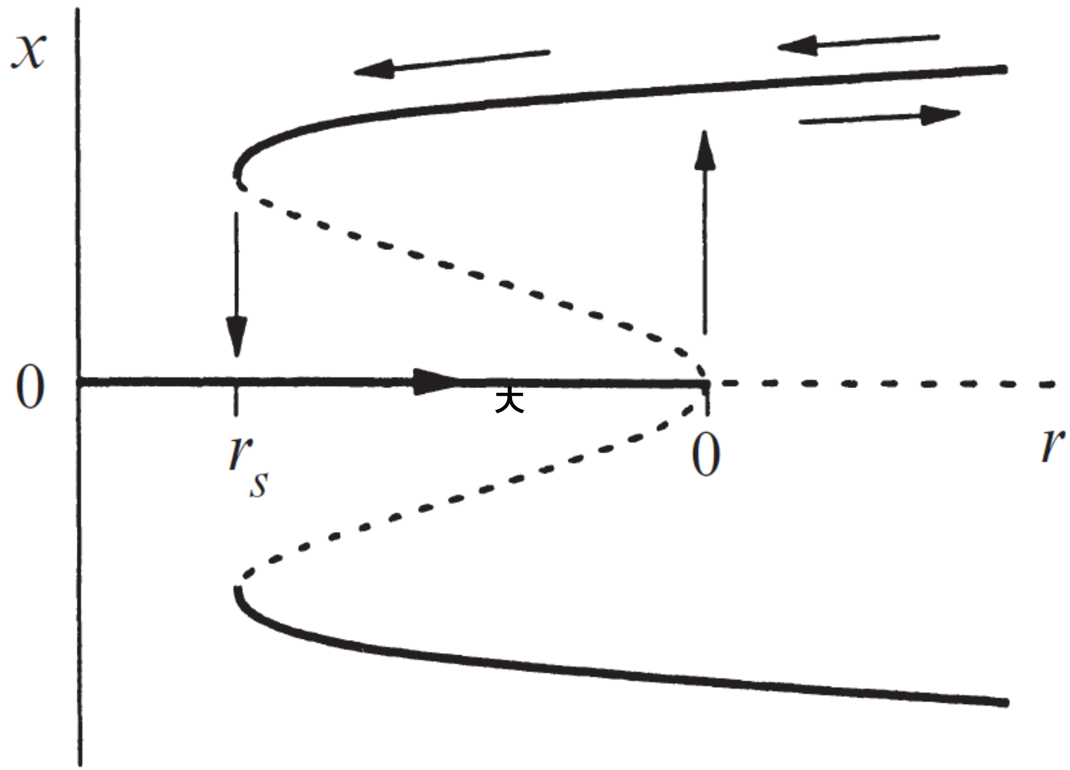
# Stabilizing term

$$\dot{x} = rx + x^3 - x^5$$




Multistability

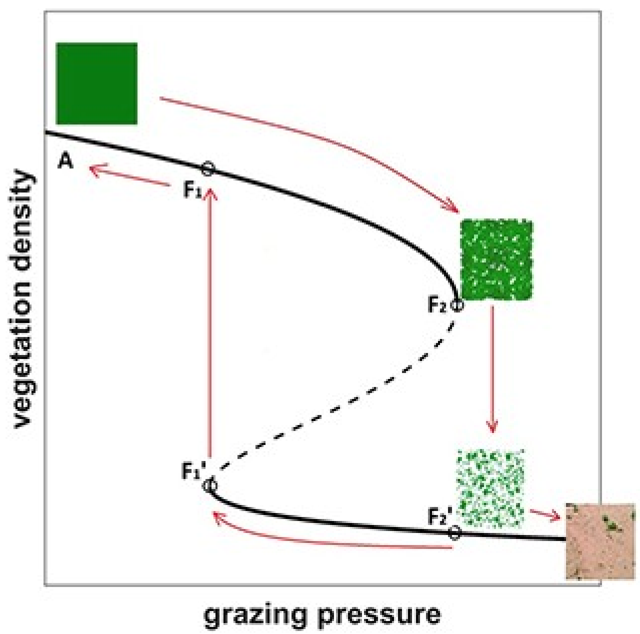
**The existence of different stable states allows for the possibility of jumps and hysteresis as the control parameter is varied.**

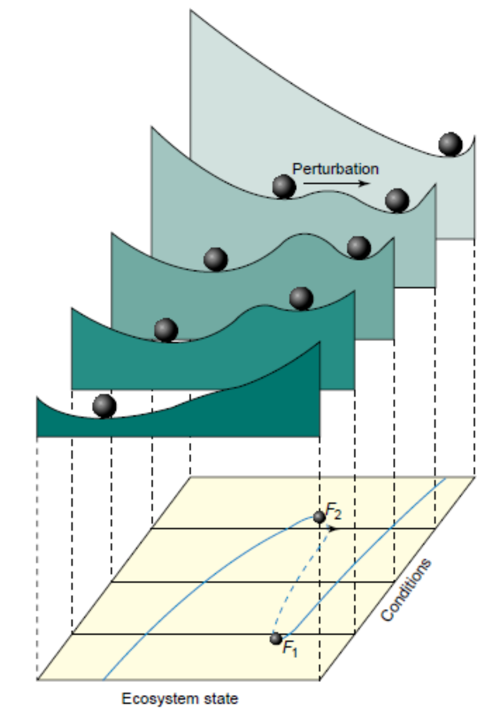


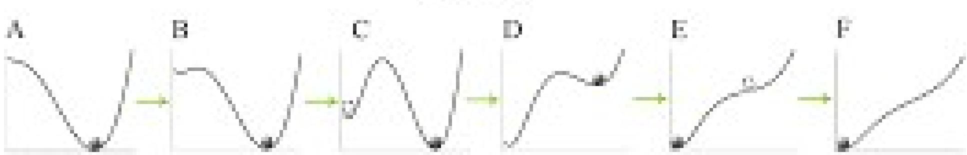

Suppose we start the system in the fixed point x*=0 and slowly increase *r*, then we jump to a stable state. If *r* is decreased the state remain on the stable branch even when r is decreased below 0.  We have to lower r down rs to get the state back to the origin. This lack of reversibility as parameter is varied is called hysteresis.

Dangerous transition

The most standard scenario for a discontinuos transition appears when we have two saddle-nodes:




$dx/dt=a+bx-x^3$

$b>0$

**Q: Do all bifurcations imply abrupt changes in the variables?**
**A: No.**

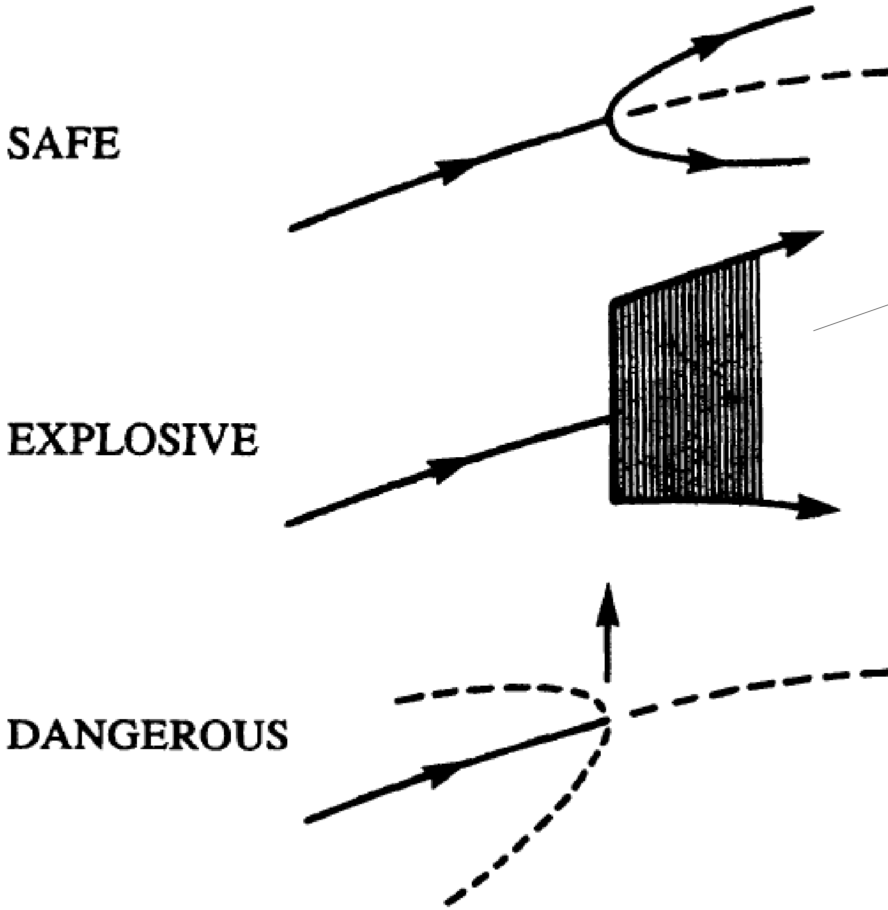
## Classifying bifurcations according to their consequences:

Thompson, Siebert, IJBC 21, 399 (2011).

1. **Safe transitions**: imply a **mild** and **reversible** change in the behavior of the system. There are **no discontinuities** in the responses characterized by the bifurcation diagram (although slopes may be discontinuous). The system exhibits **neither intermittency nor hysteresis.**

2. **Explosive transitions**: imply a **sudden transition** to another type of regime, in which the **attractor enlarges abruptly (discontinuity)**. The transition is however **reversible** by sweeping back the control parameter. Typically, **intermittent** behavior is observed, as the attractor consists on two different but connected pieces corresponding to the pre- and post-explosive regimes, and the system switches between the two qualitatively different behaviors.

3. **Dangerous transitions**: imply a **sudden jump** and at the same time **irreversible** behavior. Thus thresholds for these transitions are **points of no-return.** Consequently, the system exhibits **hysteresis**, and also **multistability**, since there are parameter ranges for which more than one regime is possible.



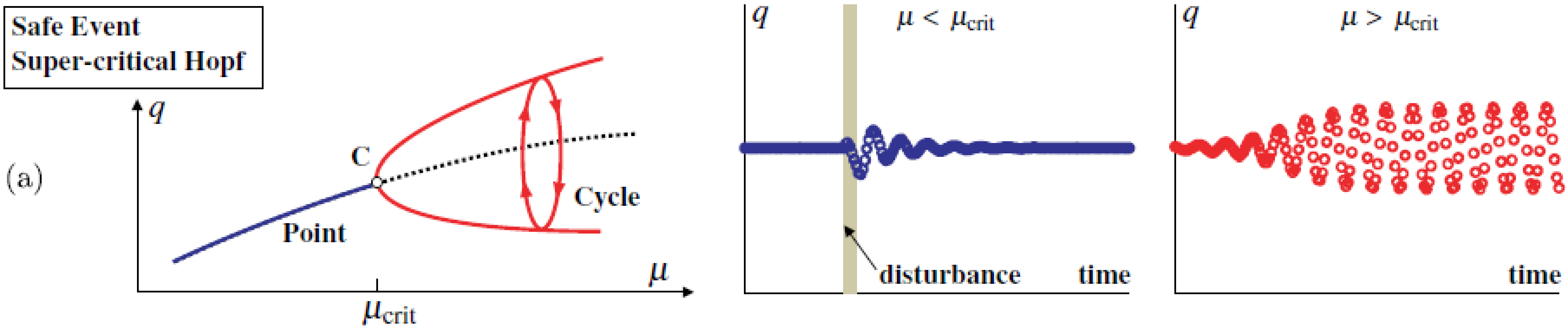

No hysteresis

Discontinuous change in size and form of the attractor

The attractor (in a discontinuos way) dissapears and the system jumps to a new attractor.

Hysteresis

# Safe bifurcations




**Safe Bifurcations**

| | | |
|---|---|---|
| (a) | **Local Supercritical Bifurcations** | |
| 1. | Supercritical Hopf | Point to cycle |
| 2. | Supercritical Neimark-Sacker (secondary Hopf) | Cycle to torus |
| 3. | Supercritical Flip (period-doubling) | Cycle to cycle |
| (b) | **Global Bifurcations** | |
| 4. | Band Merging | Chaos to chaos |

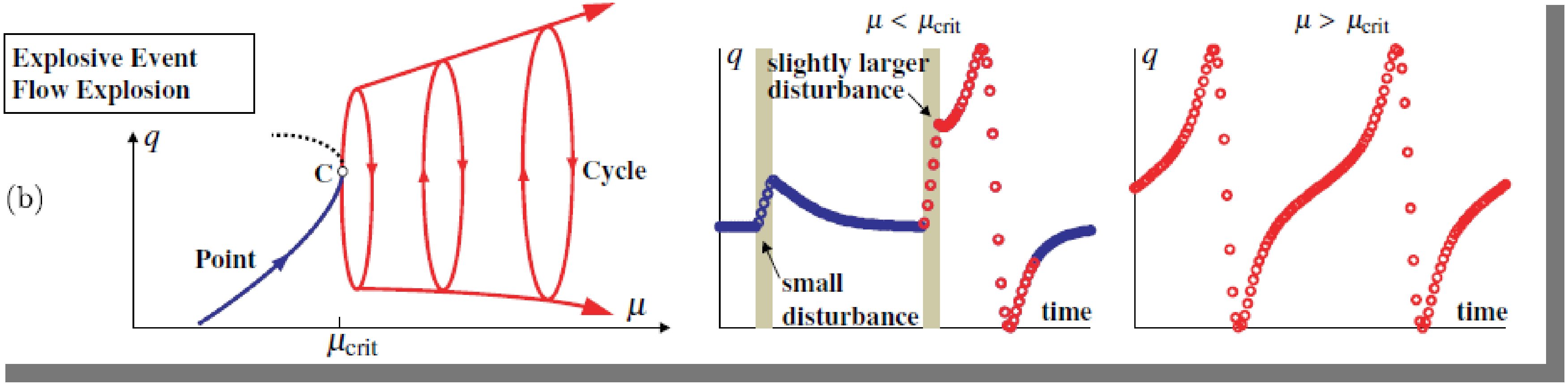
These bifurcations are characterized by the following features:

**SUBTLE**: continuous supercritical growth of new attractor path
**SAFE**: no fast jump or enlargement of the attracting set
**DETERMINATE**: single outcome even with small noise
**NO HYSTERESIS**: path retraced on reversal of control sweep
**NO BASIN CHANGE**: basin boundary remote from attractors
**NO INTERMITTENCY**: in the responses of the attractors

Also sup. pitchfork, transcritical, …

From Thompson and Sieber, IJBC 21, 399 (2011)

# Explosive bifurcations




**Explosive Bifurcations**

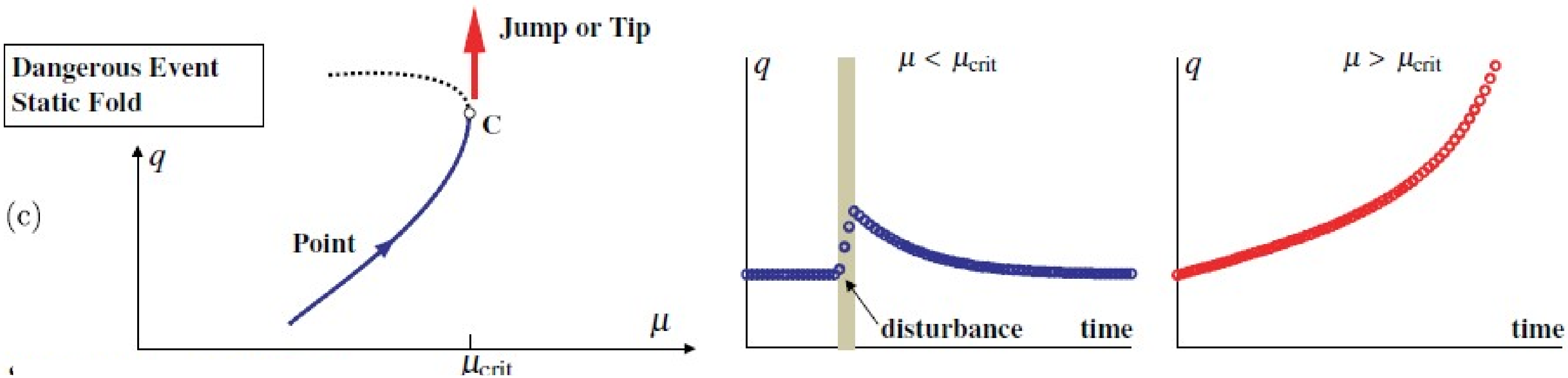
| | | |
|---|---|---|
| 5. | Flow Explosion (omega explosion, SNIPER) | Point to cycle |
| 6. | Map Explosion (omega explosion, mode-locking) | Cycle to torus |
| 7. | Intermittency Explosion: Flow | Point to chaos |
| 8. | Intermittency Explosion: Map (temporal intermittency) | Cycle to chaos |
| 9. | Regular-Saddle Explosion (interior crisis) | Chaos to chaos |
| 10. | Chaotic-Saddle Explosion (interior crisis) | Chaos to chaos |

These bifurcations are characterized by the following features:

**CATASTROPHIC**: global events, abrupt enlargement of attracting set
**EXPLOSIVE**: enlargement, but no jump to remote attractor
**DETERMINATE**: with single outcome even with small noise
**NO HYSTERESIS**: paths retraced on reversal of control sweep
**NO BASIN CHANGE**: basin boundary remote from attractors
**INTERMITTENCY**: lingering in old domain, flashes through the new

From Thompson and Sieber, IJBC 21, 399 (2011)

# Dangerous bifurcations




**Dangerous Bifurcations**

**(a) Local Saddle-Node Bifurcations**

| | | |
|---|---|---|
| 11. | Static Fold (saddle-node of fixed point) | from Point |
| 12. | Cyclic Fold (saddle-node of cycle) | from Cycle |

**(b) Local Subcritical Bifurcations**

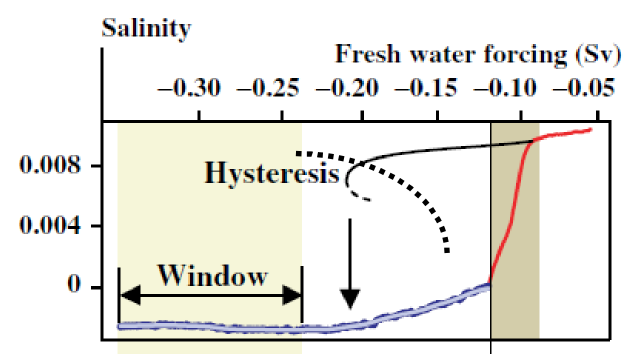
| | | |
|---|---|---|
| 13. | Subcritical Hopf | from Point |
| 14. | Subcritical Neimark-Sacker (secondary Hopf) | from Cycle |
| 15. | Subcritical Flip (period-doubling) | from Cycle |

**(c) Global Bifurcations**

| | | |
|---|---|---|
| 16. | Saddle Connection (homoclinic connection) | from Cycle |
| 17. | Regular-Saddle Catastrophe (boundary crisis) | from Chaos |
| 18. | Chaotic-Saddle Catastrophe (boundary crisis) | from Chaos |

These bifurcations are characterized by the following features:

CATASTROPHIC: sudden disappearance of attractor
DANGEROUS: sudden jump to new attractor (of any type)
INDETERMINACY: outcome can depend on global topology
HYSTERESIS: path not reinstated on control reversal
BASIN: tends to zero (b), attractor hits edge of residual basin (a, c)
NO INTERMITTENCY: but critical slowing in global events




From Thompson and Sieber, IJBC 21, 399 (2011)

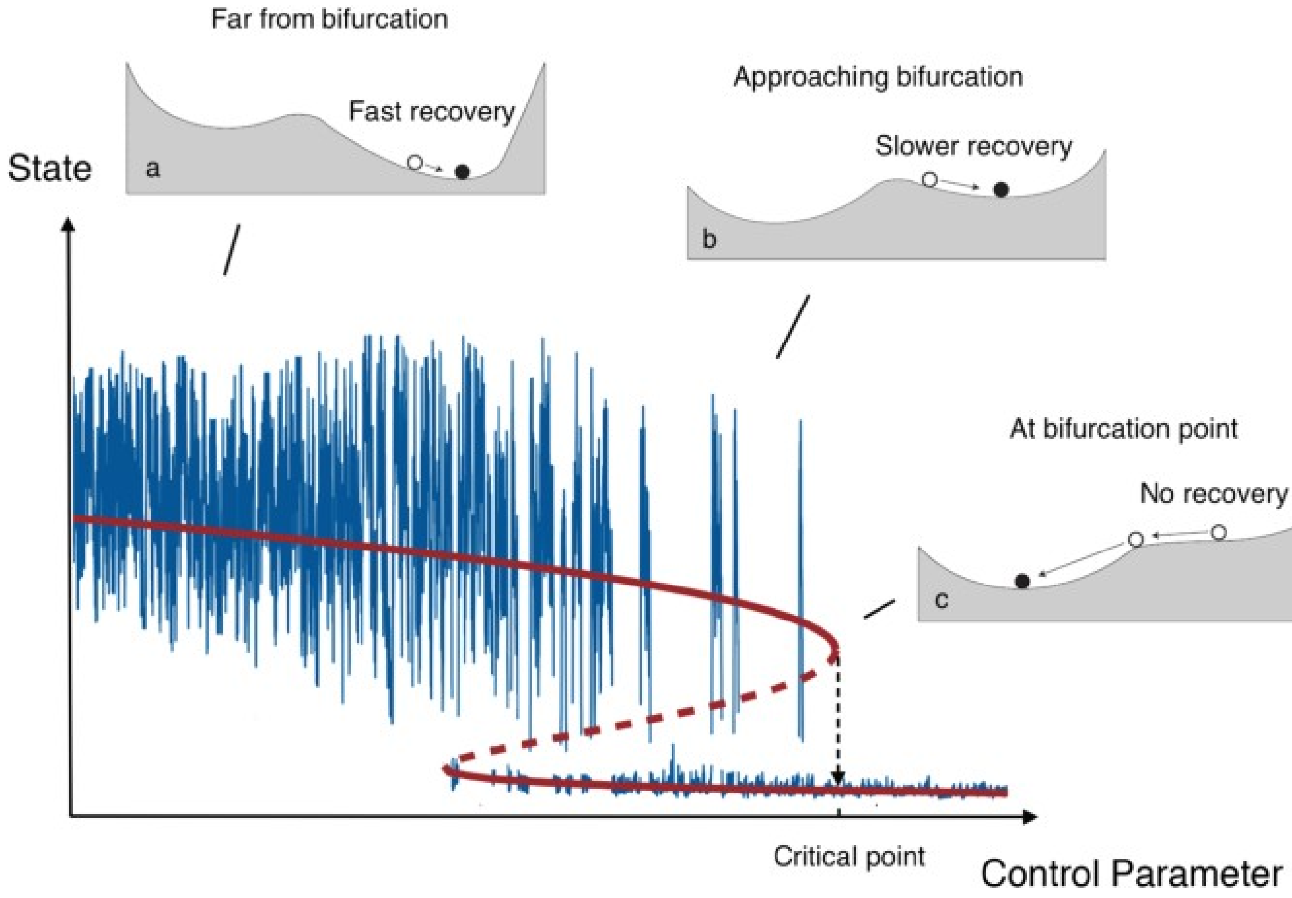
# Anticipating tipping points

- A working scientific hypothesis is that abrupt climatic transitions are associated to dangerous and to explosive bifurcations

- Since climatic abrupt transitions may have important impact on human activities (without time to slowly adapt, as with safe bifurcations), it is relevant to devise methods of early warning: is there something in the system behavior before the bifurcation telling us that we are close to a tipping point?

- Bifurcation-type tipping points carry generic early warning signals. If a system is subject to some source of small short-term fluctuations from which it tries to recover because it is an attractor, then the rate of recovery slows down as the system is steadily forced toward a bifurcation-type tipping point. **CRITICAL SLOWING DOWN.**

This critical slowing down behavior is a result of the negative feedback that maintains the attractor state weakening in strength, and positive feedback progressively taking over. It applies to nearly all bifurcations, whether they are catastrophic or not.



## Take-home messages:

- Climate transitions have occurred in the past and are likely to occur again.
- It is highly desirable to have early warning indicators which will announce us on the danger of a close transition
- Identifying tipping elements for subseasonal (short time scales) climate phenomena. Objective of **CAFE**.
- Need of anticipating tipping points: Next lectures by Emilio.