Causality and Information Transfer in Systems with Extreme Events

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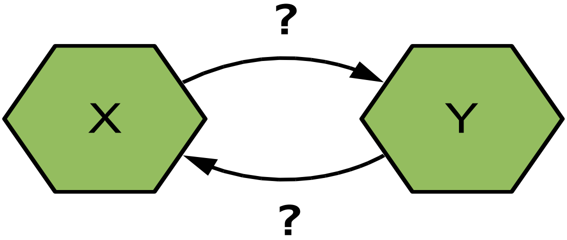
Interactions in complex systems



- EVOLUTION OF (SUB)SYSTEMS — recorded time series
- INTERACTIONS — dependence in time series
  - COUPLING — statistical dependence
  - SYNCHRONIZATION — functional relation
  - CAUSALITY — predictability

General, nonlinear dependence

random variables x, y, PDF $p(x)$, $p(y)$

independence $p(x, y) = p(x)p(y)$

digression from independence: $\log \frac{p(x,y)}{p(x)p(y)}$

a measure of general statistical dependence
MUTUAL INFORMATION

$$I(X; Y) = \sum_x \sum_y p(x, y) \log \frac{p(x, y)}{p(x)p(y)}$$

Mutual information

- mutual information

$$I(X;Y) = \sum_x \sum_y p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$$

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

- average amount of common information, contained in the variables X and Y
- measure of general statistical dependence
- $I(X;Y) \geq 0$
- $I(X;Y) = 0$ iff X and Y are independent

Conditional mutual information

- conditional mutual information $I(X;Y|Z)$ of variables X, Y given the variable Z

$$I(X;Y|Z) = H(X|Z) + H(Y|Z) - H(X,Y|Z)$$

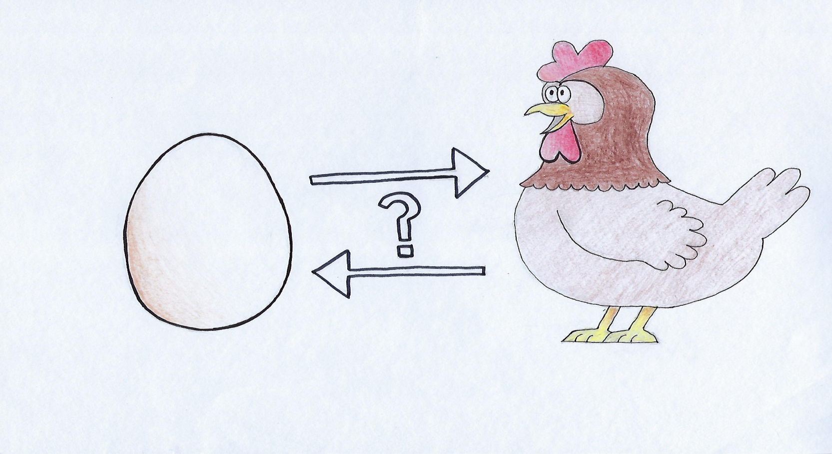
- can be expressed as

$$I(X;Y|Z) = H(X,Z) + H(Y,Z) - H(X,Y,Z) - H(Z)$$

- for Z independent of X and Y

$$I(X;Y|Z) = I(X;Y)$$

- “net” dependence between X and Y without possible influence of Z



Can we identify causes of observed phenomena?

Granger causality

C. Granger, 2003 Nobel prize in economy

- causal variable can help to forecast the effect variable after other data has been first used
- generalization using information theory
  conditional mutual information (transfer entropy)
- $I(y(t); x(t+\tau)|x(t), x(t-\eta), \ldots, x(t-(n-1)\eta))$
  - $y(t)$ – the cause (predictor)
  - $x(t+\tau)$ – the effect (the future of the influenced variable)
  - $x(t), x(t-\eta), \ldots, x(t-(n-1)\eta)$ – condition – removes the influence of history of the influenced variable
- for Gaussian systems CMI $\equiv$ TE $\equiv$ Granger causality
- causality interpreted as information transfer

Entropy definitions

random variable X with sets of values Ξ and PDF's $p(x)$

- Shannon entropy

$$H(X) = -\sum_{x\in\Xi} p(x)\log p(x)$$

- **Rényi entropy**

$$H_\alpha(X) = \frac{1}{1-\alpha}\log\sum_{x\in\Xi} p(x)^\alpha,$$

  where $\alpha > 0$, $\alpha \neq 1$.

- As $\alpha \to 1$, $H_\alpha(X)$ converges to $H(X)$ which is Shannon entropy.
- Rényi's measure satisfies $H_\alpha(x) \leq H_{\alpha'}(x)$ for $\alpha > \alpha'$.

Numerical example

- cause variable $C(t)$

$$C(t) = a_c C(t-1) + \sigma_c \xi_c(t)$$

- cause variable $X(t)$

$$X(t) = \xi_x(t)$$

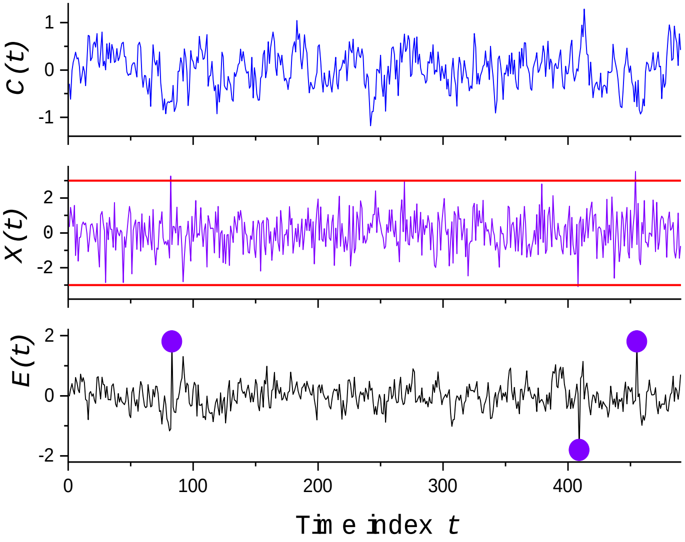
- effect variable $E(t)$

$$E(t) = a_e E(t-1) + b_e C(t-1) + \sigma_e \xi_e(t)$$

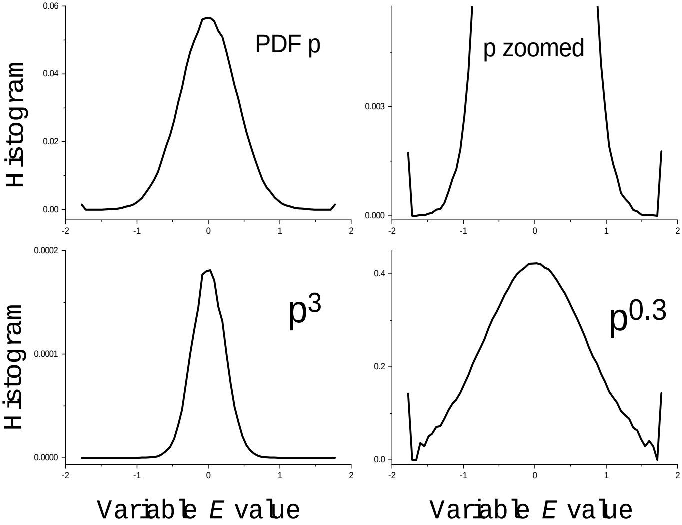
and if $X(t-1) > 3$ then $E(t) = 1.8$
and if $X(t-1) < -3$ then $E(t) = -1.8$

where $a_C = 0.7$, $\sigma_C^2 = 0.1$, $a_E = 0.5$, $b_E = 0.2$, $\sigma_E^2 = 0.1$,
ξ_C, ξ_X and ξ_E are Gaussian random variable with zero mean and unit variance

Numerical example: time series



Numerical example: histograms



Rényi entropy

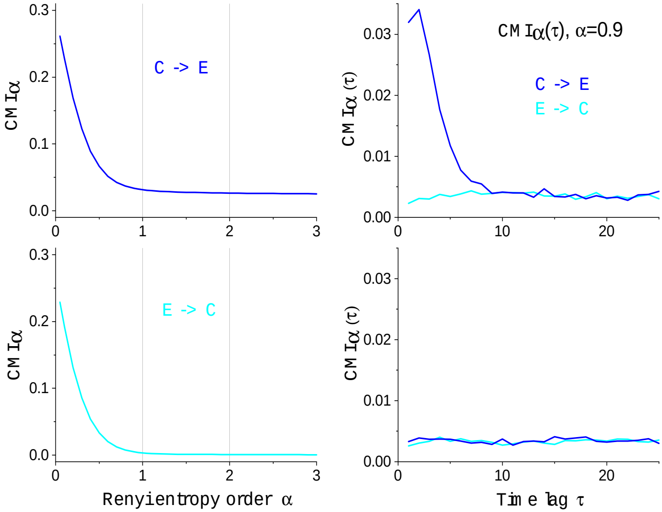
Rényi order parameter $\alpha > 0$, $\alpha \neq 1$

$$p(x)^\alpha$$

- $\alpha > 1$ "amplifies" center of PDF
- $0 < \alpha < 1$ "amplifies" **tails** of PDF
- tails – extreme events

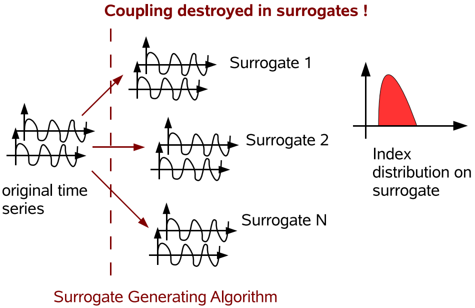
Can conditional mutual information (transfer entropy) in Rényi concept uncover causes of extreme events?

Renyi conditional mutual information, RCMI



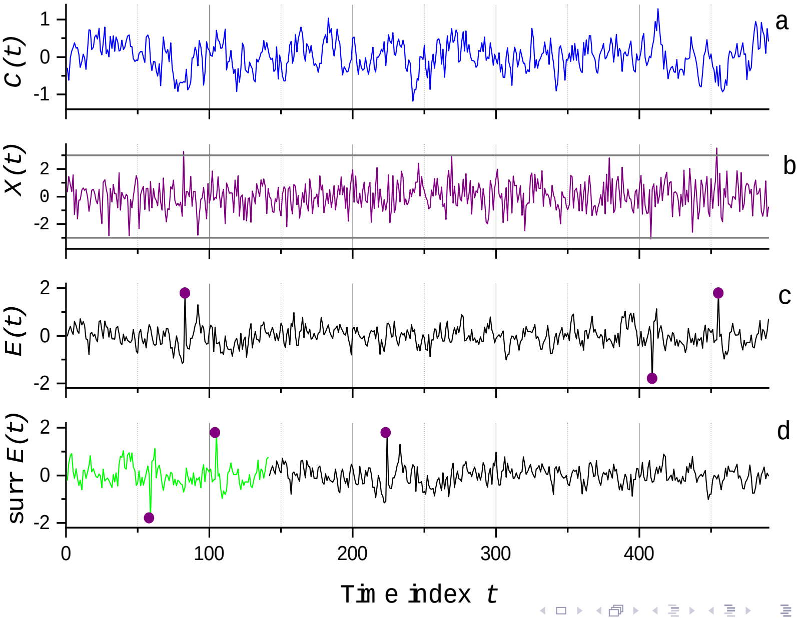
Significance testing using surrogate data

- Use of bootstrap-like strategy (surrogate time series)
- Ideally preserve all properties except tested (causality)

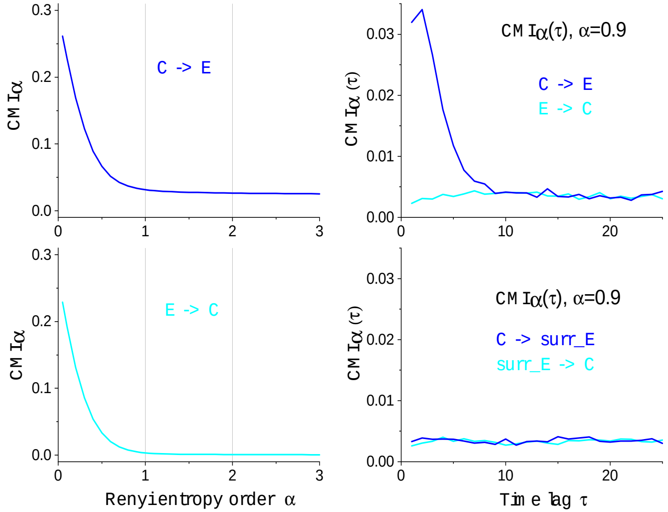


- Randomization algorithms: circular shift
- Significance of digression from the surrogate distribution
- Z-score = (data_value - surrogate_mean)/surrogate_SD

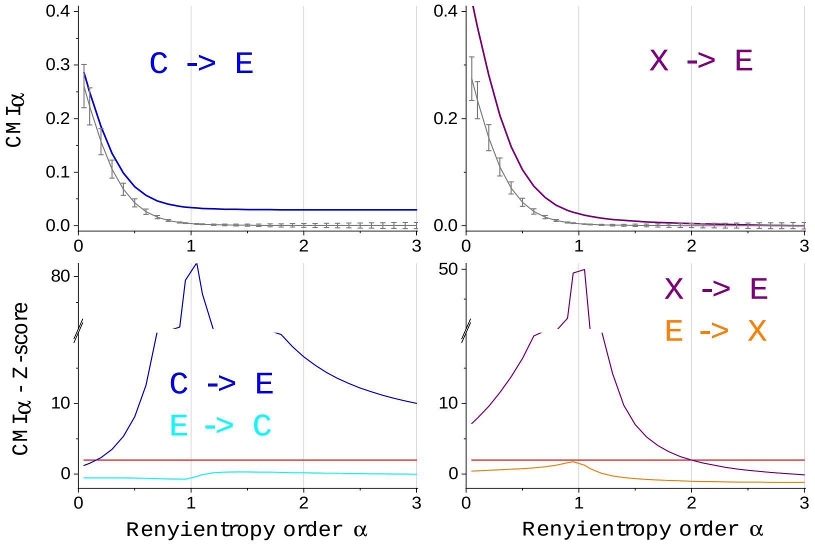
Numerical example: time series



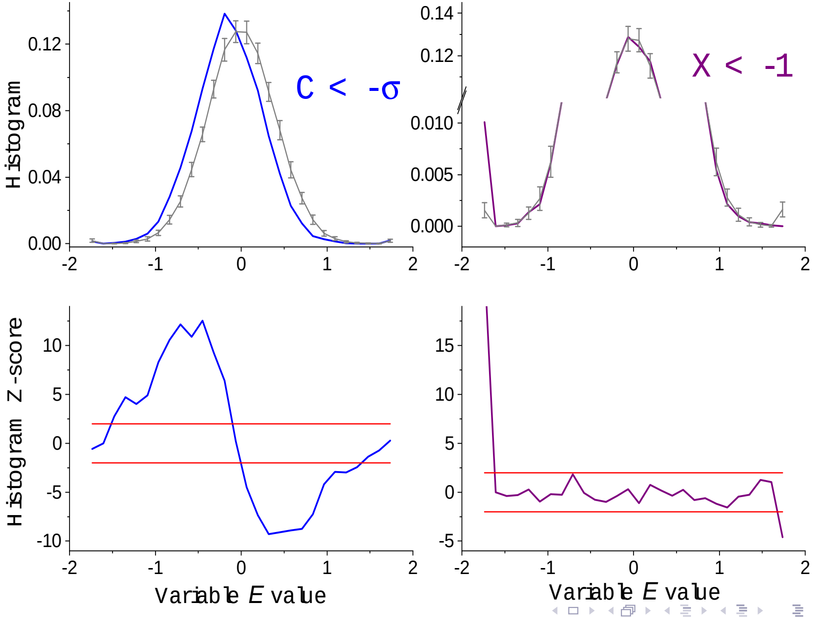
Renyi conditional mutual information, RCMI



RCMI: statistical significance results



Conditional histograms

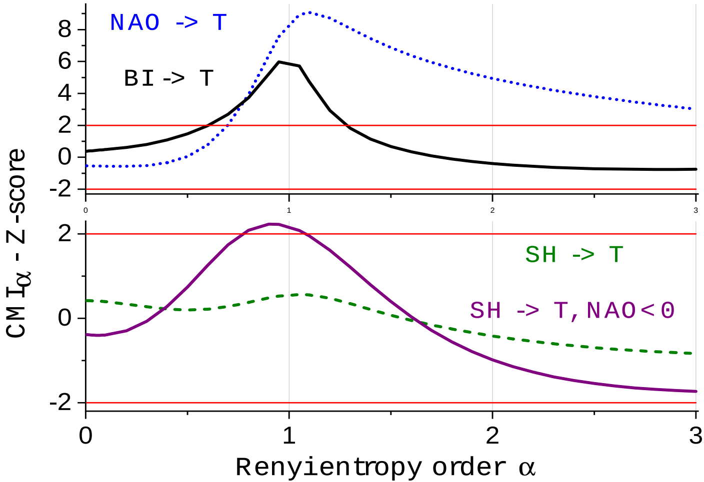


Experiment with Rényi CMI on climate data

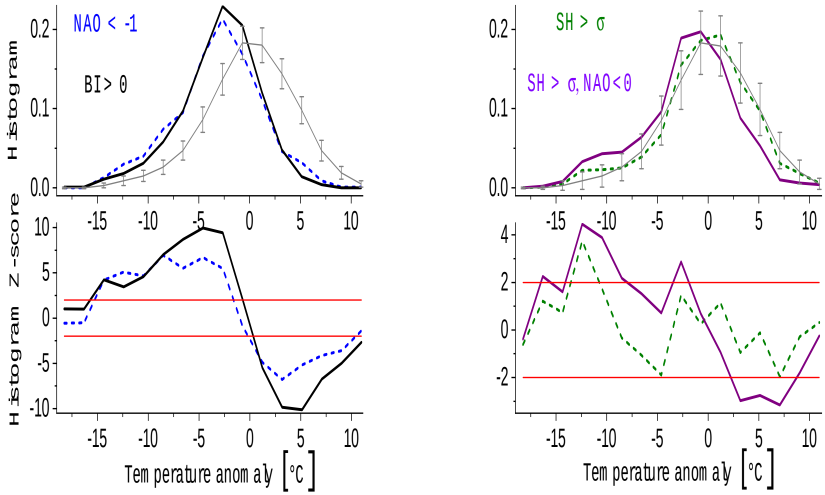
DATA

- near surface air temperature (SAT) anomalies
  - winter daily data Frankfurt (50° 02' 47"N, 8° 35' 54"E, 112 m above sea level)
  - spring daily data Dijon (47° 16' 04"N, 5° 5' 17"E, 219 m above sea level)
- daily NAO index
- daily Blocking Index
  Tibaldi S, Molteni F (1990) On the operational predictability of blocking. Tellus 42A : 343—365
- daily Siberian High

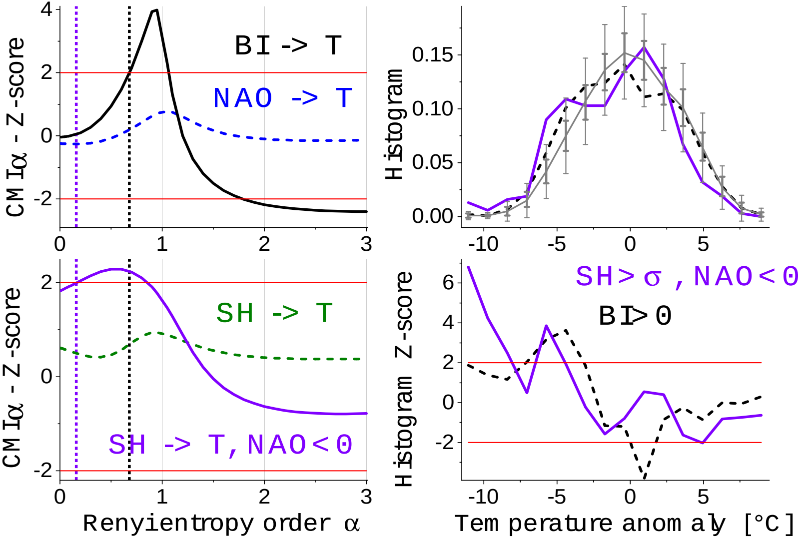
Experiment with Rényi CMI: winter Frankfurt SATA



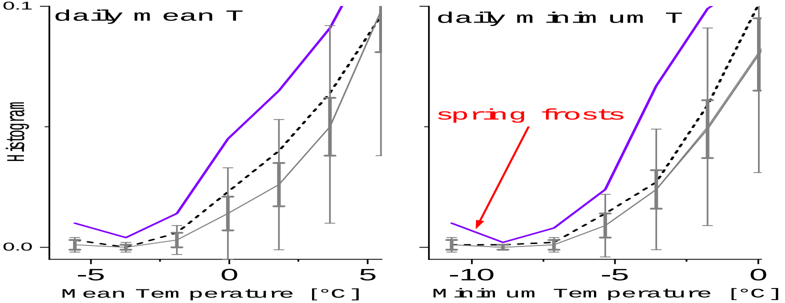
Conditional histograms: winter Frankfurt SATA



Experiment with Rényi CMI: spring Dijon SATA



Spring Dijon cold SAT extremes



RESULTS

- NAO/blocking in winter shifts means/whole histogram
  negative NAO/positive BI: colder winter, with possible cold extremes
- Siberian High ($> \sigma$) causes left tail higher than normal, under the condition of non-positive NAO
  positive NAO opens a gate for warmer air from Atlantic
  Siberian High $> \sigma$ and NAO< 0:
  higher probability of cold extremes, esp. in spring
- Information-theoretic approach to causality
  in Rényi concept seems promising
  in identification of causes of extreme events
- research in progress, publication in preparation

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Thank you for your attention

Interested in postdoc, PhD position?

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