A simple stochastic model of tropical waves

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with Samuel Stechmann (UW)

Weather and Climate Extremes and their Predictability

September 28, 2022

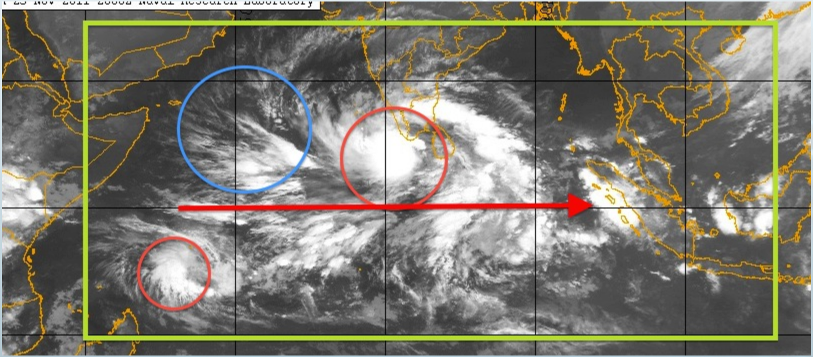
The Madden-Julian Oscillation

"The MJO occurs mainly over the remote tropical oceans and was not discovered until the early 1970s. It strongly influences precipitation over southern Asia and northern Australia, affecting the lives of **literally billions of people.** *It is also believed to influence the timing and intensity of El Niños. Despite its importance, the MJO is perhaps the last type of weather system for which the* **basic physical mechanisms are not well understood.**"

(from Randall 2012)

What is an MJO

Madden-Julian Oscillation

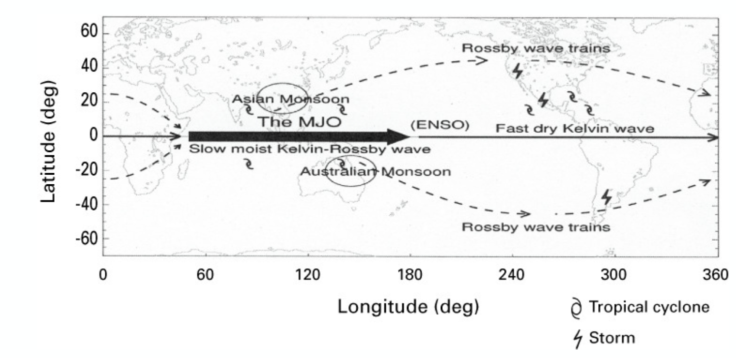


Courtesy A. Adames

- Black lines = 15° = 1650 km $\approx$ 1025 miles!
- Storm is roughly 6600 km in length.
- Circumference of Earth $\approx$ 40,000 km.

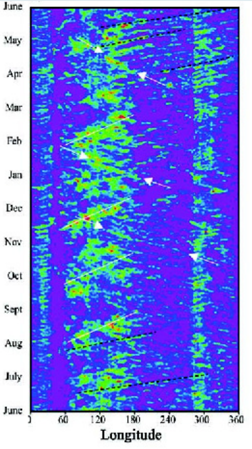
Global Impact

- El-Niño
- Tropical Cyclones
- Monsoons
- Predictability in the Mid-lattitudes



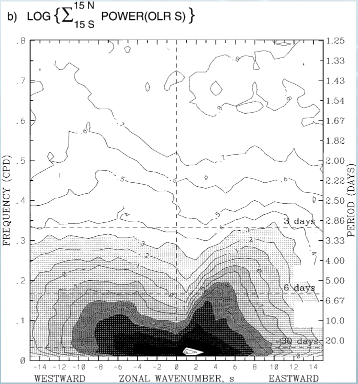
Multiscale clouds and waves

Precipitation



2000–2001 (from Zhang 2005)

Spectral Power
(of Fourier transform in space & time)

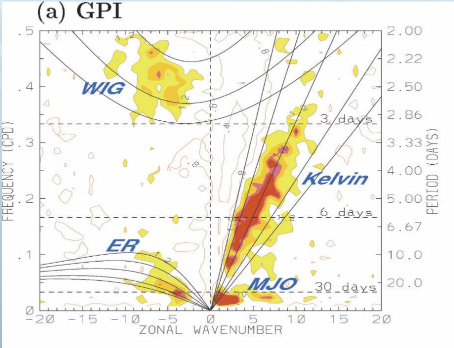


from Wheeler & Kiladis 1999

Goal

Develop a unified, simple model of both

1. The Madden-Julian Oscillation (MJO)
2. Convectively Coupled Equatorial Waves (CCEWs)



Prior Models

1. MJO
   - Boundary-layer frictional convergence
     - (Wang & Rui, 1990; Salby et al. 1994)
   - Moisture mode theories
     - (Raymond & Fuchs, 2009; Sobel & Maloney, 2013; Adames & Kim, 2016)
   - Skeleton Model
     - (Majda & Stechmann, 2009, 2011)
2. CCEWs
   - Convective adjustment and quasi-equilibrium theory
     - (Gill, 1982; Emanuel et al., 1994; Neelin and Zeng, 2000)
   - More complex models that include multiple cloud types
     - Stratiform-(Mapes, 2000)
     - Congestus-(Khouider and Majda, 2006, 2007, 2008)

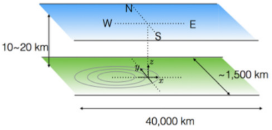
Waves of the tropical atmosphere

Convectively coupled equatorial waves: mathematical theory

Linearized 3D equations for rotating hydrostatic Boussinesq fluid

$$\partial_t u - \beta y v = -\partial_x p + S^u,$$
$$\partial_t v + \beta y u = -\partial_y p + S^v,$$
$$g\frac{\theta}{\theta_{ref}} = \partial_z p,$$
$$\partial_t \theta + w\frac{d\theta_{bg}}{dz} = S^\theta,$$
$$\partial_x u + \partial_y v + \partial_z w = 0.$$

(u, v, w)	Winds in the (x, y, z) direction
p	pressure
θ	potential temperature
βy	Coriolis force - equatorial β-plane approximation
S^u, S^v, S^θ	forcing terms
θ_{ref}	reference potential temperature
$\frac{d\theta_{bg}}{dz}$	background potential temperature gradient
g	acceleration due to gravity



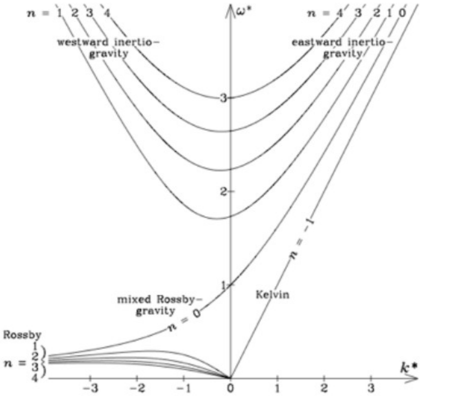
Credit: HR Ogrosky (VCU)

Simplified Equations

Starting point:

Kelvin wave	Mixed Rossby-gravity (MRG) waves	Rossby and inertia-gravity (IG) waves
$\partial_t r_0 + \partial_x r_0 = 0$	$\partial_t r_1 + \partial_x r_1 - v_0 = 0,$ $\partial_t v_0 + r_1 = 0$	$\partial_t r_{m+1} + \partial_x r_{m+1} - \sqrt{m+1} v_m = 0,$ $\partial_t l_{m-1} - \partial_x l_{m-1} - \sqrt{m} v_m = 0,$ $\partial_t v_m + \sqrt{m+1} r_{m+1} + \sqrt{m} l_{m-1} = 0.$

Dispersion relation for each wave type



How to couple moisture

Here we consider heat sources and moisture sinks

$$
\begin{aligned}
\partial_t u - yv - \partial_x \theta &= 0 \\
\partial_t v + yu - \partial_y \theta &= 0 \\
\partial_t \theta - \partial_x u - \partial_y v &= S^\theta \\
\partial_t q + \tilde{Q}(\partial_x u + \partial_y v) &= S^q .
\end{aligned}
$$

Our Model

$$\partial_t u - yv - \partial_x\theta = -\frac{1}{\tau_u}u$$

$$\partial_t v + yu - \partial_y\theta = -\frac{1}{\tau_v}v$$

$$\partial_t\theta - \partial_x u - \partial_y v = \frac{1}{\tau_l}q_l + \frac{1}{\tau_m}q_m - \frac{1}{\tau_\theta}\theta$$

$$\partial_t q_l + \tilde{Q}_l(\partial_x u + \partial_y v) = -\frac{1}{\tau_l}q_l + b_l\nabla^2 q_l + F_l + D_l\dot{W}_l$$

$$\partial_t q_m + \tilde{Q}_m(\partial_x u + \partial_y v) = -\frac{1}{\tau_m}q_m + b_m\nabla^2 q_m + F_m + D_m\dot{W}_m$$

$q_{\text{low/mid}}$-Lower/Middle tropospheric moisture,

- Two vertical levels of moisture with convective adjustment time scales $\tau_{low} = 1\text{h}$, and $\tau_{mid} = 1.33$ days.

Advantages

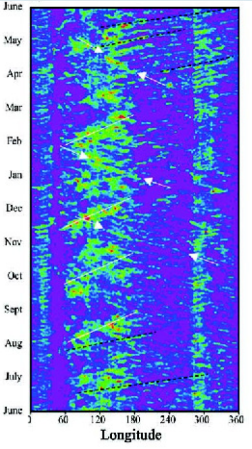
- The model is a linear SPDE!
- In Fourier space (x)

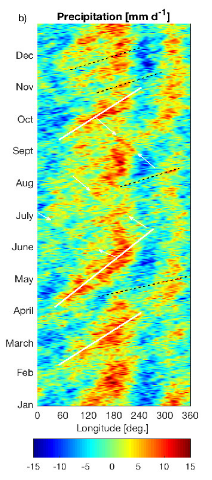
$$\frac{d\boldsymbol{U}(k,t)}{dt} = \boldsymbol{A}_k \boldsymbol{U}(k,t) + \boldsymbol{D}\dot{\boldsymbol{W}}(k,t) + \boldsymbol{F}(k,t).$$

- Equation is solved exactly for any time t and wavenumber k.
- Only error is the sampling of the space-time white noise.
- All statistics are solved exactly.

Results

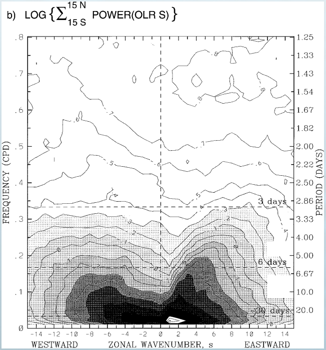
Hovmollers



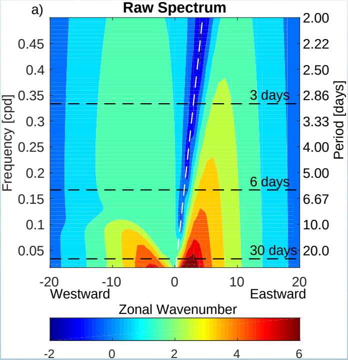


(from Zhang 2005)

Results: PSD-Raw

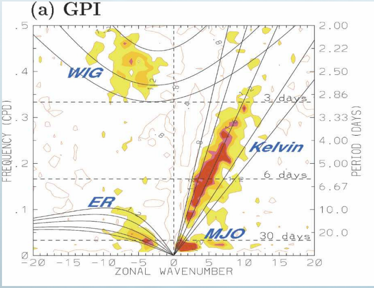


from Wheeler & Kiladis 1999

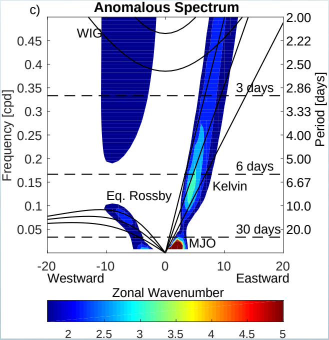


Model

Results: PSD-Anamolies



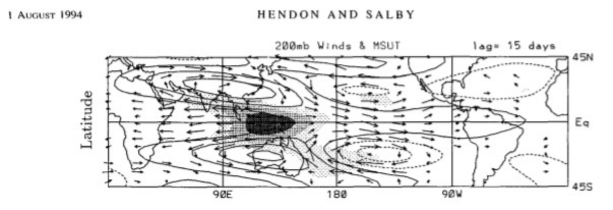
(from Lin et al. 2006)



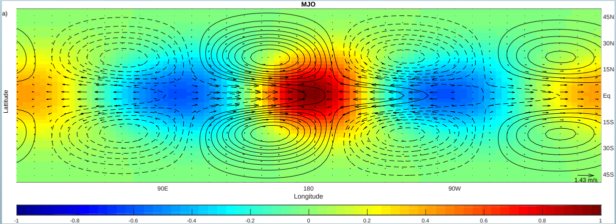
Model

Results: 2D Structure

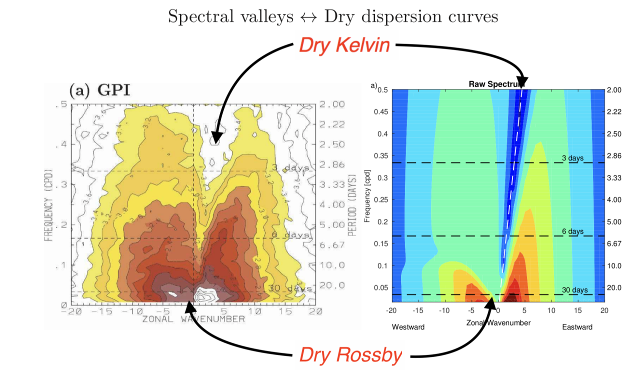
Structure (Lag/Lead)



(from Hendon & Salby 1994)



Mechanism for suppression of a wave?



Courtesy, Samuel Stechmann

- Spectral valleys caused by anti-resonance?

Anti-Resonance Mechanism

Coupled system: dry variables $(\mathbf{u}, \theta) \leftrightarrow$ water vapor (q)

Idealization as ODEs:

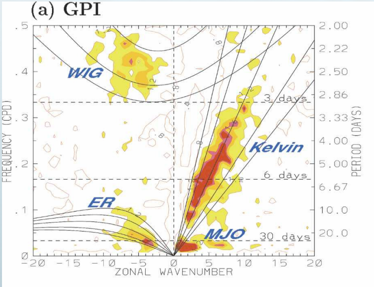
$$\frac{d^2 u}{dt^2} + \omega_d^2 u = q \qquad \text{Dry dynamics, with convective heating}$$

$$\frac{d^2 q}{dt^2} + a u = F_0 \cos \omega_f t \qquad \text{Moisture dynamics, with wave coupling}$$

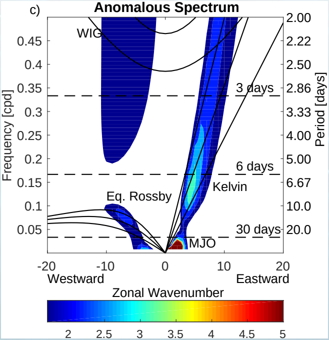
Anti-resonant forcing $\Rightarrow$ $q = 0, \quad u \propto \cos \omega_f t$

$\omega_f = \omega_d$ $\Rightarrow$ spectral *valley* at $\omega = \omega_d$

Mechanism for suppression of a wave?



(from Lin et al. 2006)



Model

- *Since rainfall acts as a heat source for the dry dynamical core, rainfall oscillations are suppressed at the natural frequencies of the dry dynamics.*

Summary

A new, simple, unified model for the MJO and CCEWs.

- Just convective adjustment and eddy diffusion of moisture.
- Key: two convective adjustment time scales τ_{low} and τ_{mid}.
- Anti-resonance a cause for spectral valleys and suppression of waves?

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Thank you for your attention!